PET ENGINEERING COLLEGE

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DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

UNIT – I

SYSTEM COMPONENTS & THEIR REPRESENTATION

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SUBJECT CODE : EC3351

SUBJECT NAME : CONTROL SYSTEM

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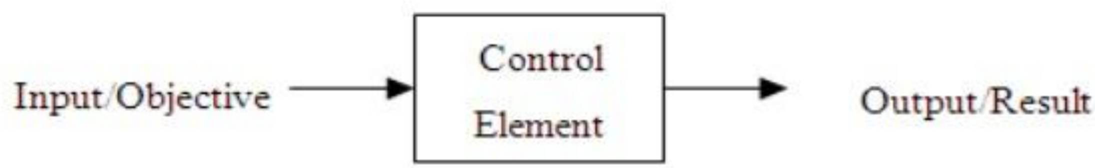
UNIT- I
SYSTEM COMPONENTS & THEIR REPRESENTATION

Basic elements of control system:

In recent years, control systems have gained an increasingly importance in the development and advancement of the modern civilization and technology. Figure shows the basic components of a control system. Disregard the complexity of the system; it consists of an input (objective), the control system and its output (result). Practically our day-to-day activities are affected by some type of control systems. There are two main branches of control systems:

1) Open-loop systems and

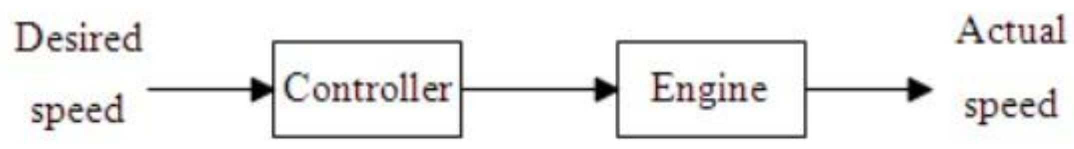
2) Closed-loop systems.



Basic Components of Control System

1. Open-loop systems:

The open-loop system is also called the non-feedback system. This is the simpler of the two systems. A simple example is illustrated by the speed control of an automobile as shown in Figure 1-2. In this open-loop system, there is no way to ensure the actual speed is close to the desired speed automatically. The actual speed might be way off the desired speed because of the wind speed and/or road conditions, such as uphill or downhill etc.



Basic Open Loop System

Practical Examples of Open Loop Control System

1. Electric Hand Drier - Hot air (output) comes out as long as you keep your hand under the machine, irrespective of how much your hand is dried.
2. Automatic Washing Machine - This machine runs according to the pre-set time irrespective of washing is completed or not.
3. Bread Toaster - This machine runs as per adjusted time irrespective of toasting is completed or not.
4. Automatic Tea/Coffee Maker - These machines also function for pre adjusted time only.
5. Timer Based Clothes Drier - This machine dries wet clothes for pre-adjusted time, it does not matter how much the clothes are dried.
6. Light Switch - Lamps glow whenever light switch is on irrespective of light is required or not.
7. Volume on Stereo System - Volume is adjusted manually irrespective of output volume level.

Advantages of Open Loop Control System

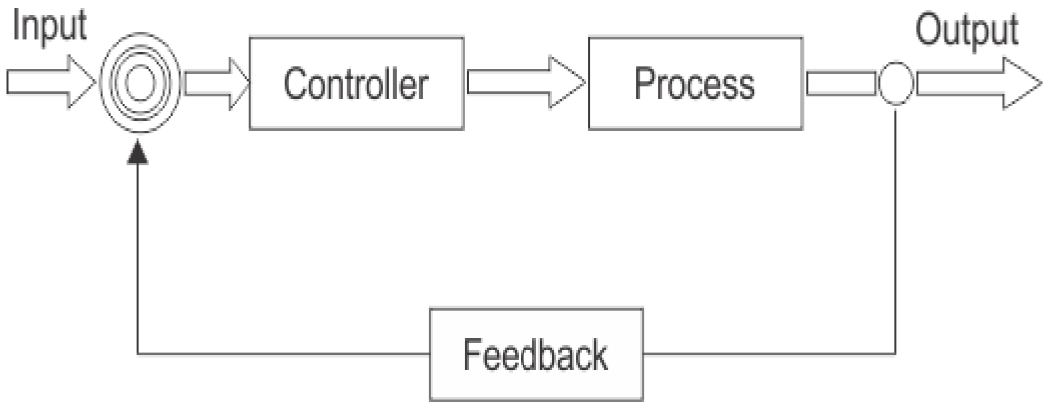
1. Simple in construction and design.
2. Economical.
3. Easy to maintain.
4. Generally stable.
5. Convenient to use as output is difficult to measure.

Disadvantages of Open Loop Control System

1. They are inaccurate.
2. They are unreliable.
3. Any change in output cannot be corrected automatically.

2. Closed-loop systems:

The closed-loop system is also called the feedback system. A simple closed-system is shown in Figure 1-3. It has a mechanism to ensure the actual speed is close to the desired speed automatically.



Practical Examples of Closed Loop Control System

1. Automatic Electric Iron - Heating elements are controlled by output temperature of the iron.
2. Servo Voltage Stabilizer - Voltage controller operates depending upon output voltage of the system.
3. Water Level Controller - Input water is controlled by water level of the reservoir.
4. Missile Launched and Auto Tracked by Radar - The direction of missile is controlled by comparing the target and position of the missile.
5. An Air Conditioner - An air conditioner functions depending upon the temperature of the room.
6. Cooling System in Car - It operates depending upon the temperature which it controls.

Advantages of Closed Loop Control System

1. Closed loop control systems are more accurate even in the presence of non-linearity.
2. Highly accurate as any error arising is corrected due to presence of feedback signal.
3. Bandwidth range is large.
4. Facilitates automation.
5. The sensitivity of system may be made small to make system more stable.
6. This system is less affected by noise.

Disadvantages of Closed Loop Control System

1. They are costlier.
2. They are complicated to design.
3. Required more maintenance.
4. Feedback leads to oscillatory response.
5. Overall gain is reduced due to presence of feedback.
6. Stability is the major problem and more care is needed to design a stable closed loop system.

Comparison of Closed Loop And Open Loop Control System

Open Loop	Closed Loop
Any change in output has no effect on the input. Example : Feedback does not exists	Changes in output, affects the input which is possible by use of feedback
Output measurement is not required for operation of system	Output measurement is necessary
Feedback element is absent	Feedback element is present
Error detector is absent	Error detector is necessary
It is inaccurate and unreliable	Highly accurate and reliable
Highly sensitive to the disturbances	Less sensitive to the disturbances
Highly sensitive to the environmental changes	Less sensitive to the environmental changes
Bandwidth is small	Bandwidth is large
Simple to construct and cheap	Complicated to design and hence costly
Generally are stable in nature	Stability is the major consideration while designing
Highly affected by non-linearity	Reduced effect of nonlinearities

Electrical Analogies of Mechanical Systems

Two systems are said to be **analogous** to each other if the following two conditions are satisfied.

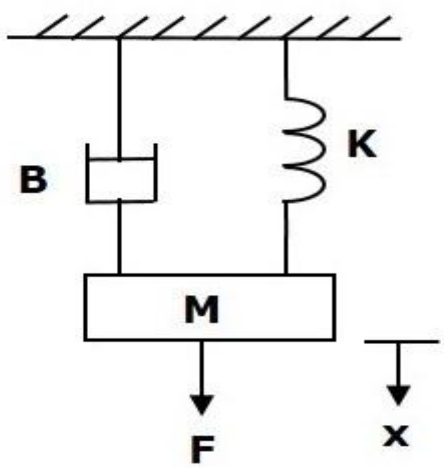
- The two systems are physically different
- Differential equation modelling of these two systems are same

Electrical systems and mechanical systems are two physically different systems. There are two types of electrical analogies of translational mechanical systems. Those are force voltage analogy and force current analogy.

Force Voltage Analogy

In force voltage analogy, the mathematical equations of **translational mechanical system** are compared with mesh equations of the electrical system.

Consider the following translational mechanical system as shown in the following figure.



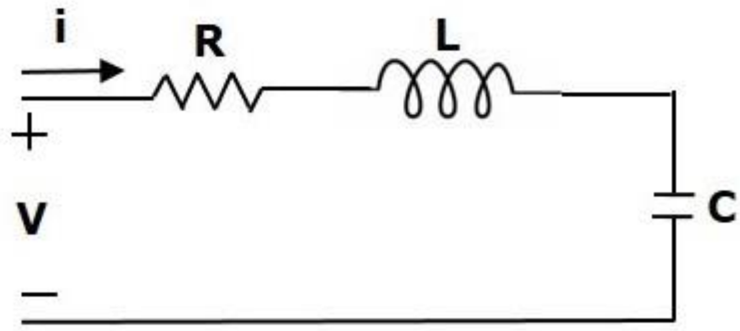
The **force balanced equation** for this system is

$$F = F_m + F_b + F_k$$

$$\Rightarrow F = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx$$

(Equation 1)

Consider the following electrical system as shown in the following figure. This circuit consists of a resistor, an inductor and a capacitor. All these electrical elements are connected in a series. The input voltage applied to this circuit is VV volts and the current flowing through the circuit is ii Amps.



Mesh equation for this circuit is

$$V = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{1}{c}\int i dt$$ **(Equation 2)**

Substitute, $i = \frac{\mathrm{d}q}{\mathrm{d}t}$ in Equation 2.

$$V = R\frac{\mathrm{d}q}{\mathrm{d}t} + L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + \frac{q}{C}$$

$$\Rightarrow V = L\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + R\frac{\mathrm{d}q}{\mathrm{d}t} + \left(\frac{1}{c}\right)q$$

(Equation 3)

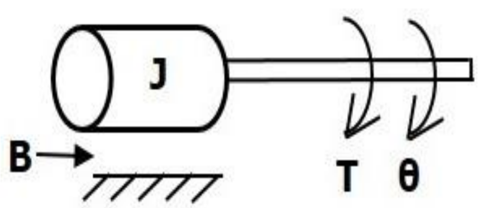
By comparing Equation 1 and Equation 3, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)

Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance (1c)(1c)
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Torque Voltage Analogy:

In this analogy, the mathematical equations of **rotational mechanical system** are compared with mesh equations of the electrical system. Rotational mechanical system is shown in the following figure.



The torque balanced equation is

$$T = T_j + T_b + T_k$$

$$\Rightarrow T = J\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + B\frac{\mathrm{d}\theta}{\mathrm{d}t} + k\theta$$

(Equation 4)

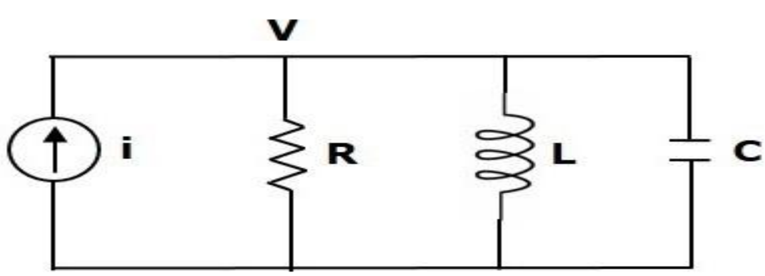
By comparing Equation 4 and Equation 3, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Voltage(V)
Moment of Inertia(J)	Inductance(L)
Rotational friction coefficient(B)	Resistance(R)
Torsional spring constant(K)	Reciprocal of Capacitance (1c)(1c)
Angular Displacement(θ)	Charge(q)
Angular Velocity(ω)	Current(i)

Force Current Analogy:

In force current analogy, the mathematical equations of the **translational mechanical system** are compared with the nodal equations of the electrical system.

Consider the following electrical system as shown in the following figure. This circuit consists of current source, resistor, inductor and capacitor. All these electrical elements are connected in parallel.



The nodal equation is

$$i=\frac{V}{R}+\frac{1}{L}\int V dt+C\frac{\mathrm{d}V}{\mathrm{d}t}$$ **(Equation 5)**

Substitute, $V=\frac{\mathrm{d}\Psi}{\mathrm{d}t}$ in Equation 5.

$$i=\frac{1}{R}\frac{\mathrm{d}\Psi}{\mathrm{d}t}+\left(\frac{1}{L}\right)\Psi+C\frac{\mathrm{d}^2\Psi}{\mathrm{d}t^2}$$

$$\Rightarrow i=C\frac{\mathrm{d}^2\Psi}{\mathrm{d}t^2}+\left(\frac{1}{R}\right)\frac{\mathrm{d}\Psi}{\mathrm{d}t}+\left(\frac{1}{L}\right)\Psi$$ **(Equation 6)**

By comparing Equation 1 and Equation 6, we will get the analogous quantities of the translational mechanical system and electrical system. The following table shows these analogous quantities.

Translational Mechanical System	Electrical System
Force(F)	Current(i)
Mass(M)	Capacitance(C)
Frictional coefficient(B)	Reciprocal of Resistance(1R)(1R)
Spring constant(K)	Reciprocal of Inductance(1L)(1L)
Displacement(x)	Magnetic Flux(ψ)
Velocity(v)	Voltage(V)

Similarly, there is a torque current analogy for rotational mechanical systems.

Torque Current Analogy:

In this analogy, the mathematical equations of the **rotational mechanical system** are compared with the nodal mesh equations of the electrical system.

By comparing Equation 4 and Equation 6, we will get the analogous quantities of rotational mechanical system and electrical system. The following table shows these analogous quantities.

Rotational Mechanical System	Electrical System
Torque(T)	Current(i)
Moment of inertia(J)	Capacitance(C)
Rotational friction coefficient(B)	Reciprocal of Resistance(1R)(1R)
Torsion spring constant(K)	Reciprocal of Inductance(1L)(1L)
Angular displacement(θ)	Magnetic flux(ψ)
Angular velocity(ω)	Voltage(V)

Transfer Function of Control System

A control system consists of an output as well as an input signal. The output is related to the input through a function call **transfer function**.

Definition of Transfer Function

The transfer function of a control system is defined as the ration of the Laplace transform of the output variable to Laplace transform of the input variable assuming all initial conditions to be zero.

$$G(s) = \frac{C(s)}{R(s)}$$

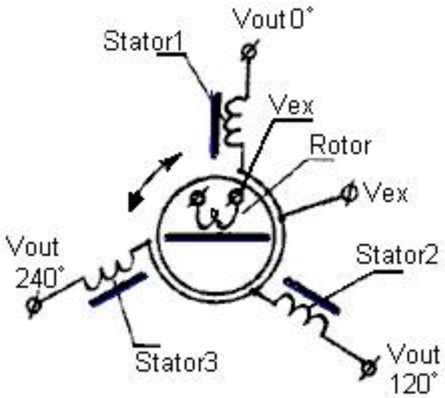
For any control system there exists a reference input termed as excitation or cause which operates through a transfer operation termed as **transfer function** and produces an effect resulting in controlled output or response. Thus the cause and effect relationship between the output and input is related to each other through a **transfer function**.

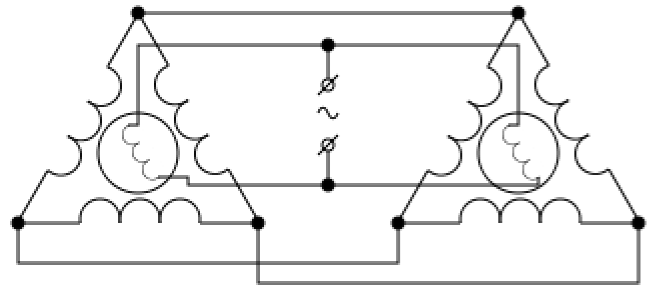
In Laplace Transform, if the input is represented by R(s) and output is represented by C(s), then the transfer function will be $G(s) = \frac{C(s)}{R(s)} \Rightarrow R(s).G(s) = C(s)$

That is, transfer function of the system multiplied by input function gives the output function of the system.

Synchro :

A **synchro** is, in effect, a transformer whose primary-to-secondary coupling may be varied by physically changing the relative orientation of the two windings. Synchros are often used for measuring the angle of a rotating machine such as an antenna platform. In its general physical construction, it is much like an electric motor. The primary winding of the transformer, fixed to the rotor, is excited by an alternating current, which by electromagnetic induction, causes currents to flow in three Y-connected secondary windings fixed at 120 degrees to each other on the stator. The relative magnitudes of secondary currents are measured and used to determine the angle of the rotor relative to the stator, or the currents can be used to directly drive a receiver synchro that will rotate in unison with the synchro transmitter. In the latter case, the whole device may be called a **selsyn** (a portmanteau of *self* and *synchronizing*).





There are two types of synchros systems: Torque systems and control systems.

In a torque system, synchros will provide a low-power mechanical output sufficient to position an indicating device, actuate a sensitive switch or move light loads without power amplification. In simpler terms, a torque synchros system is a system in which the transmitted signal does the usable work. In such a system, accuracy on the order of one degree is attainable.

Servo Motor:

Servo Motor is also called Control motors. They are used in feedback control systems as output actuators and does not use for continuous energy conversion. The principle of the Servomotor is similar to that of the other electromagnetic motor, but the construction and the operation are different. Their power rating varies from a fraction of a watt to a few hundred watts.The rotor inertia of the motors is low and have a high speed of response. The rotor of the Motor has the long length and smaller diameter. They operate at very low speed and sometimes even at the zero speed.The servo motor is widely used in radar and computers, robot, machine tool, tracking and guidance systems, processing controlling.

Classification of Servo Motor:

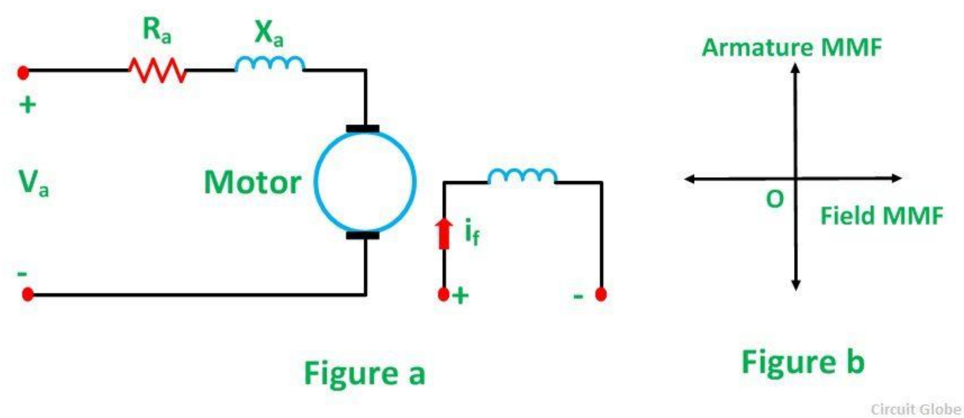
They are classified as AC and DC Servo Motor. The AC servomotor is further divided into two types.

Two Phase AC Servo Motor

Three Phase AC Servo Motor

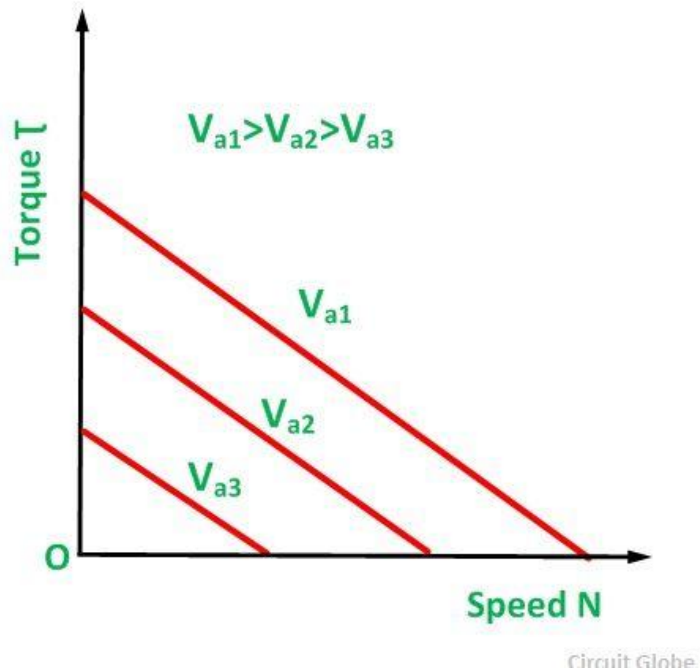
DC Servo Motor:

DC Servo Motors are separately excited DC motor or permanent magnet DC motors. The figure (a) shows the connection of Separately Excited DC Servo motor and the figure (b) shows the armature MMF and the excitation field MMF in quadrature in a DC machine.



This provides a fast torque response because torque and flux are decoupled. Therefore, a small change in the armature voltage or current brings a significant shift in the position or speed of the rotor. Most of the high power servo motors are mainly DC.

The Torque-Speed Characteristics of the Motor is shown below.



As from the above characteristics, it is seen that the slope is negative. Thus, a negative slope provides viscous damping for the servo drive system.

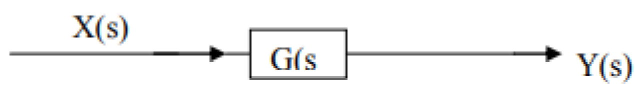
AC Servo Motor

Servo motors are generally an assembly of four things: a DC motor, a gearing set, a control circuit and a position-sensor (usually a potentiometer). The position ofservo motors can be controlled more precisely than those of standard DC motors, and they usually have three wires (power, ground & control).The AC Servo Motors are divided into two types 2 and 3 Phase AC servomotor. Most of the AC servomotor are of the two-phase squirrel cage induction motor type. They are used for low power applications. The three phase squirrel cage induction motor is now utilized for the applications where high power system is required.

BLOCK DIAGRAM

A control system may consist of a number of components. A block diagram of a system is a pictorial representation of the function performed by each component and of the flow of signals. Such a diagram depicts the inter-relationships which exists between the various components. A block diagram has the advantage of indicating more realistically the signal flows of the actual system. In a block diagram all system variables are linked to each other through functional blocks. The ―Functional Block‖ or simply ―Block‖ is a symbol for the mathematical operation on the input signal to the block which produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of flow of signals. Note that signal can pass only in the direction of arrows. Thus a block diagram of a control system explicitly shows a unilateral property.

Below Fig shows an element of the block diagram. The arrow head pointing towards the block indicates the input and the arrow head away from the block represents the output. Such arrows are entered as signals.



Block

The transfer function of a component is represented by a block. Block has single input and single output.

The following figure shows a block having input X(s), output Y(s) and the transfer function G(s).

Transfer Function,

$$G(s) = \frac{Y(s)}{X(s)}$$

$$\Rightarrow Y(s) = G(s)X(s)$$

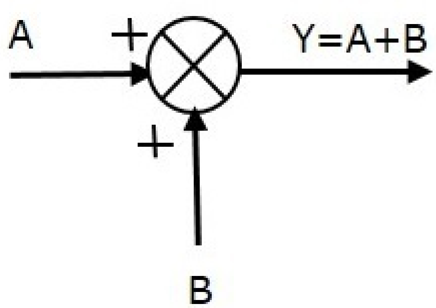
Output of the block is obtained by multiplying transfer function of the block with input.

Summing Point

The summing point is represented with a circle having cross (X) inside it. It has two or more inputs and single output. It produces the algebraic sum of the inputs. It also performs the summation or subtraction or combination of summation and subtraction of the inputs based on the polarity of the inputs. Let us see these three operations one by one.

The following figure shows the summing point with two inputs (A, B) and one output (Y). Here, the inputs A and B have a positive sign. So, the summing point produces the output, Y as **sum of A and B.**

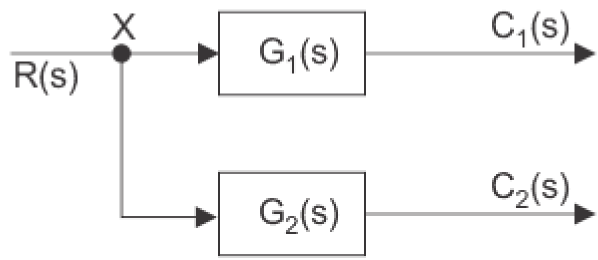
i.e.,Y = A + B.



Take-off Point

The take-off point is a point from which the same input signal can be passed through more than one branch. That means with the help of take-off point, we can apply the same input to one or more blocks, summing points.

In the following figure, the take-off point is used to connect the same input, R(s) to two more blocks.



Block Diagram Reduction Rules

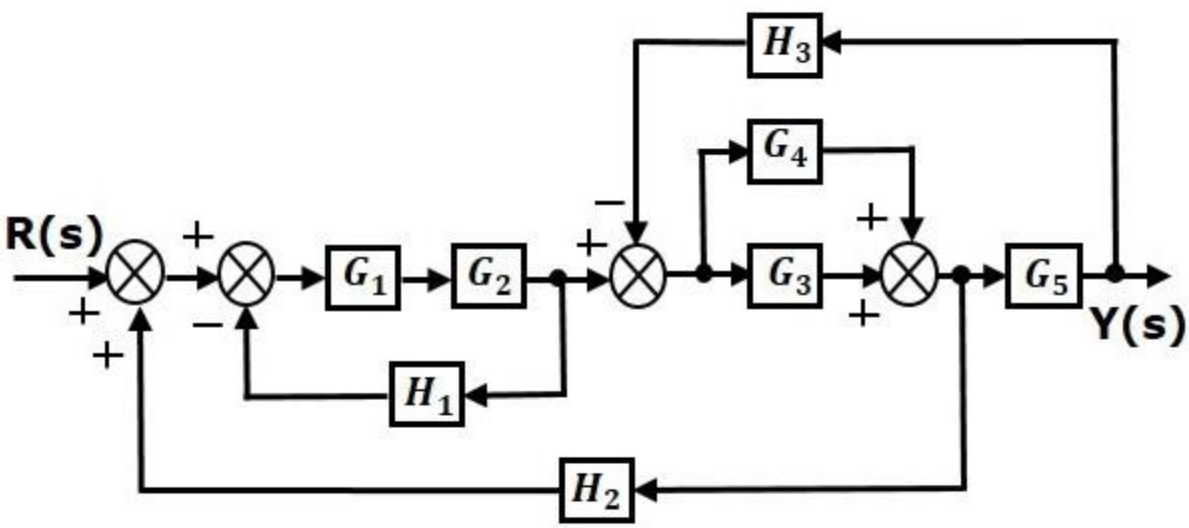
Follow these rules for simplifying (reducing) the block diagram, which is having many blocks, summing points and take-off points.

- **Rule 1 –** Check for the blocks connected in series and simplify.
- **Rule 2 –** Check for the blocks connected in parallel and simplify.
- **Rule 3 –** Check for the blocks connected in feedback loop and simplify.
- **Rule 4 –** If there is difficulty with take-off point while simplifying, shift it towards right.
- **Rule 5 –** If there is difficulty with summing point while simplifying, shift it towards left.
- **Rule 6 –** Repeat the above steps till you get the simplified form, i.e., single block.

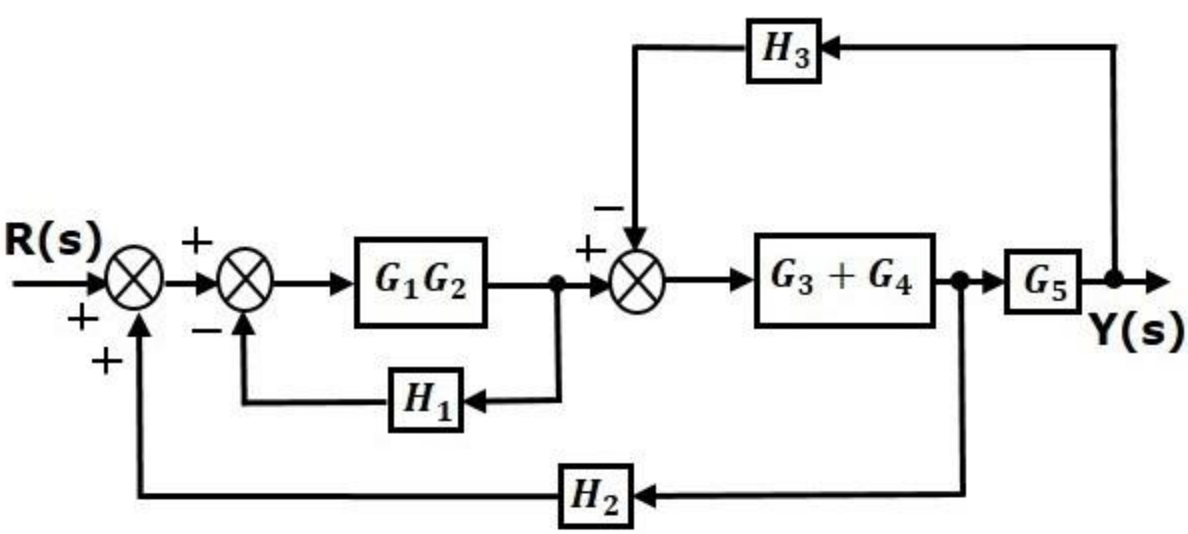
Note – The transfer function present in this single block is the transfer function of the overall block diagram.

Example

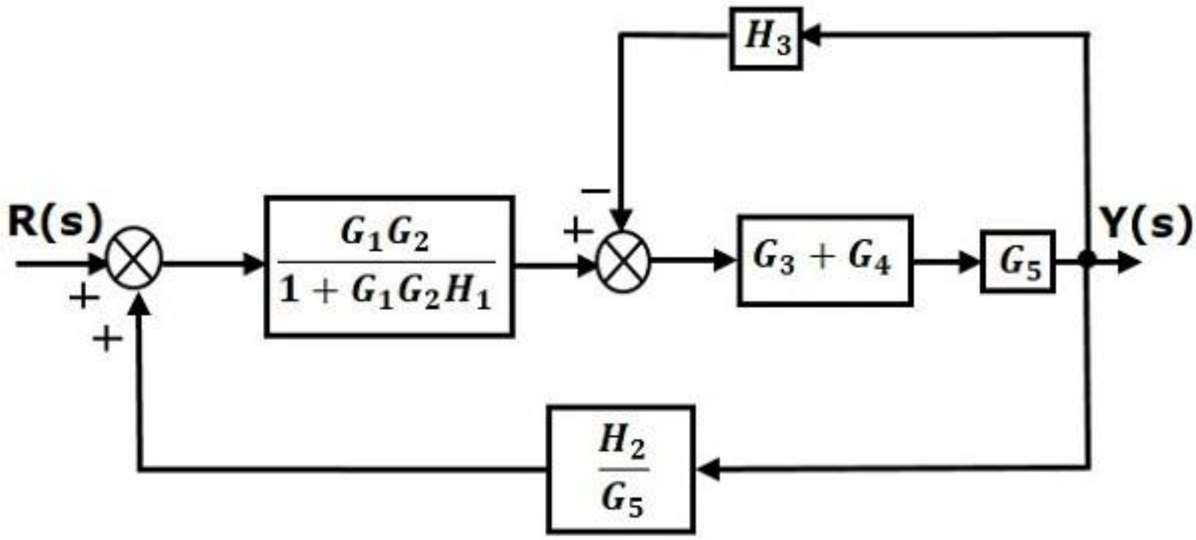
Consider the block diagram shown in the following figure. Let us simplify (reduce) this block diagram using the block diagram reduction rules.



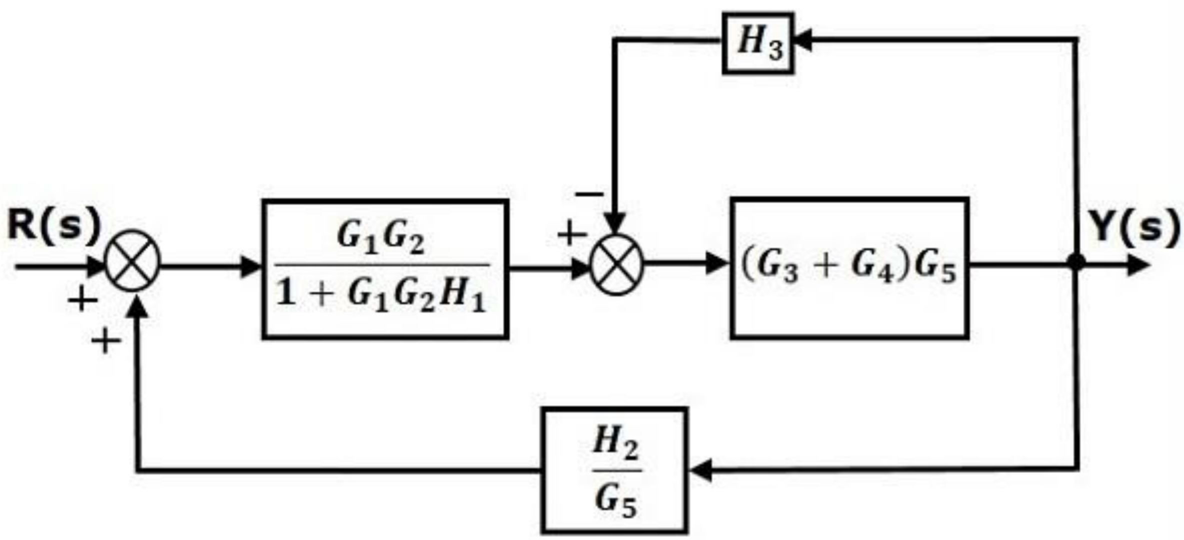
Step 1 – Use Rule 1 for blocks G1 and G2. Use Rule 2 for blocks G3 and G4. The modified block diagram is shown in the following figure.



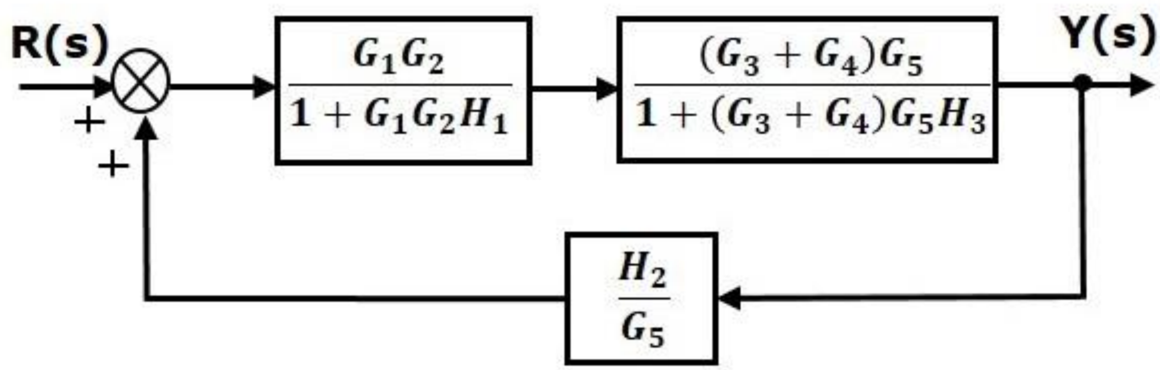
Step 2 – Use Rule 3 for blocks G1G2 and H1. Use Rule 4 for shifting take-off point after the block G5. The modified block diagram is shown in the following figure.



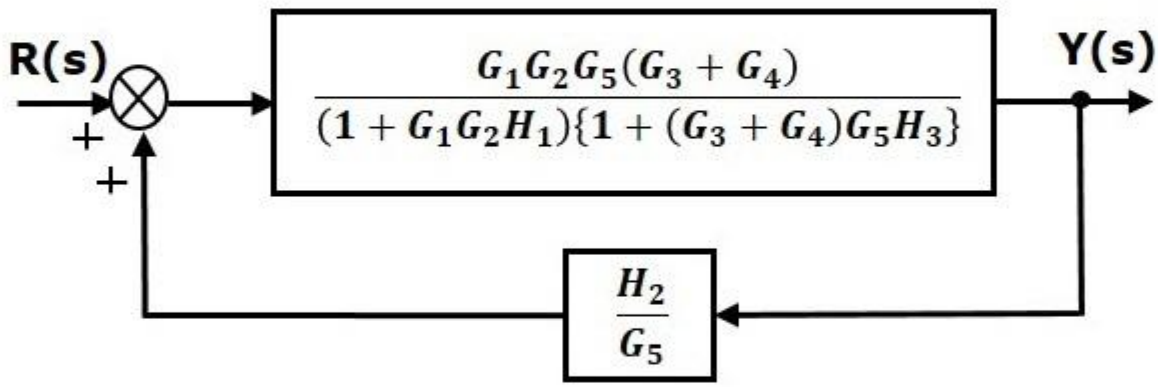
Step 3 – Use Rule 1 for blocks (G3+G4) and G5. The modified block diagram is shown in the following figure.



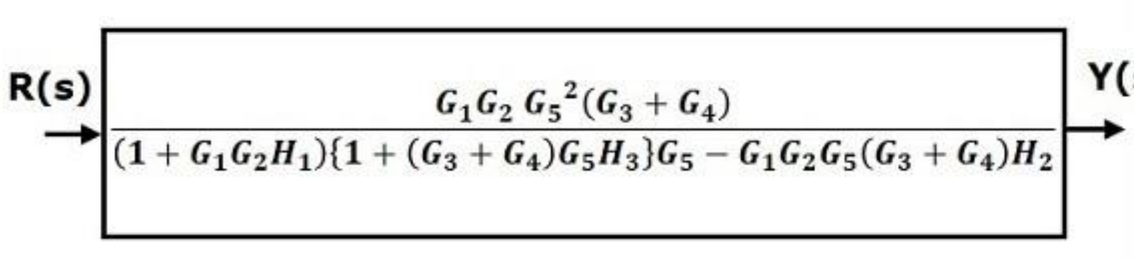
Step 4 – Use Rule 3 for blocks (G3+G4)G5 and H3. The modified block diagram is shown in the following figure.



Step 5 – Use Rule 1 for blocks connected in series. The modified block diagram is shown in the following figure.



Step 6 – Use Rule 3 for blocks connected in feedback loop. The modified block diagram is shown in the following figure. This is the simplified block diagram.



Therefore, the transfer function of the system is

Y(s)R(s)=G1G2G25(G3+G4)(1+G1G2H1){1+(G3+G4)G5H3}G5−G1G2G5(G3+G4)H2Y(s)R(s)=G1G2G52(G3+G4)(1+G1G2H1){1+(G3+G4)G5H3}G5−G1G2G5(G3+G4)H2

Note – Follow these steps in order to calculate the transfer function of the block diagram having multiple inputs.

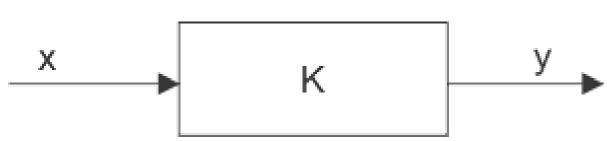
- **Step 1 –** Find the transfer function of block diagram by considering one input at a time and make the remaining inputs as zero.
- **Step 2 –** Repeat step 1 for remaining inputs.
- **Step 3 –** Get the overall transfer function by adding all those transfer functions.

Signal Flow Graph of Control System

The block diagram reduction process takes more time for complicated systems. Because, we have to draw the (partially simplified) block diagram after each step. So, to overcome this drawback, use signal flow graphs (representation).

In the next two chapters, we will discuss about the concepts related to signal flow graphs, i.e., how to represent signal flow graph from a given block diagram and calculation of transfer function just by using a gain formula without doing any reduction process.

Signal flow graph of control system is further simplification of block diagram of control system. Here, the blocks of transfer function, summing symbols and take off points are eliminated by branches and nodes.The transfer function is referred as transmittance in signal flow graph. Let us take an example of equation y = Kx. This equation can be represented with block diagram as below.



The same equation can be represented by signal flow graph, where x is input variable node, y is output variable node and a is the transmittance of the branch connecting directly these two nodes.

Key Definitions:

- Input Node: Node with only outgoing branches;
- Output Node: Node with incoming branches. Note: Any non-input node can be made an output node by adding a branch with gain= 1.
- Path: Collection of branches linked together in same direction.
- Forward Path: Path from input node to output node where node is visited more than once.
- Gain of Forward Path: Product of all gains of branches in the forward path.
- Loop: Path that originates and terminates at the same node. No other node is visited more than once.
- Loop Gain: Product of branch gains in a loop.
- Non-Touching: Two parts of a SFG are non-touching if they do not share at least one node

Mason's gain formula is

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{N} P_i \Delta_i}{\Delta}$$

Where,

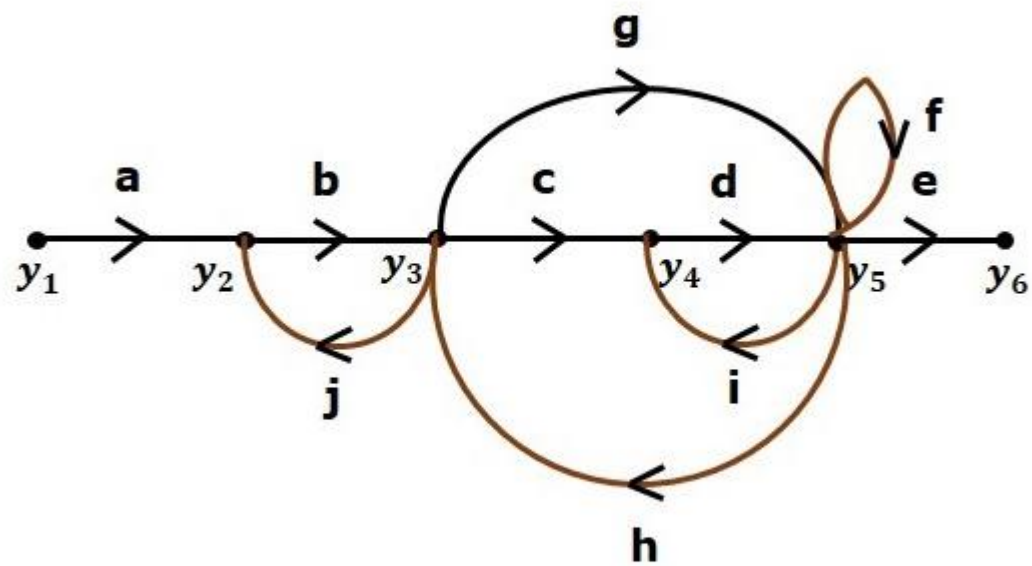
- **C(s)** is the output node
- **R(s)** is the input node
- **T** is the transfer function or gain between R(s)R(s) and C(s)C(s)
- $\mathbf{P_i}$ is the i[th] forward path gain

Δ=1−(sum of all individual loop gains) +(sum of gain products of all possible two nontouching loops) −(sum of gain products of all possible three nontouching loops)+...

Δ_i is obtained from Δ by removing the loops which are touching the i^{th} forward path.

calculation of Transfer Function using Mason's Gain Formula

Let us consider the same signal flow graph for finding transfer function.



- Number of forward paths, N = 2.
- First forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_6$
- First forward path gain, $p_1 = abcde$
- Second forward path is - $y_1 \rightarrow y_2 \rightarrow y_3 \rightarrow y_5 \rightarrow y_6$.
- Second forward path gain, $p_2 = abge$
- Number of individual loops, L = 5.
- Loops are - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_3 \rightarrow y_5 \rightarrow y_3$, $y_3 \rightarrow y_4 \rightarrow y_5 \rightarrow y_3$, $y_4 \rightarrow y_5 \rightarrow y_4$ and $y_5 \rightarrow y_5$.

- Loop gains are - $l_1=bj$, $l_2=gh$, l3=cd h, l4=di and l5=f.
- Number of two non-touching loops = 2.

- First non-touching loops pair is - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_4 \rightarrow y_5 \rightarrow y_4$.
- Gain product of first non-touching loops pair, $l_1 l_4=bjdi$l1l4=bjdi
- Second non-touching loops pair is - $y_2 \rightarrow y_3 \rightarrow y_2$, $y_5 \rightarrow y_5$.
- Gain product of second non-touching loops pair is - $l_1 l_5=bjf$

Higher number of (more than two) non-touching loops are not present in this signal flow graph.

We know,

$\Delta=1-$(sum of all individual loop gains)+(sum of gain products of all possible two non touching loops)−(sum of gain products of all possible three non touching loops)+...

Substitute the values in the above equation,

$\Delta=1-(bj+gh+cdh+di+f)+(bjdi+bjf)-(0)$

$\Rightarrow \Delta=1-(bj+gh+cdh+di+f)+bjdi+bjf$

There is no loop which is non-touching to the first forward path.

So, $\Delta_1=1$.

Similarly, $\Delta_2=1$. Since, no loop which is non-touching to the second forward path.

Substitute, N = 2 in Mason's gain formula

$$T = \frac{C(s)}{R(s)} = \frac{\Sigma_{i=1}^{2} P_i \Delta_i}{\Delta}$$

$$T = \frac{C(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Substitute all the necessary values in the above equation.

$$T = \frac{C(s)}{R(s)} = \frac{(abcde)1 + (abge)1}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

$$\Rightarrow T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$

Therefore, the transfer function is -

$$T = \frac{C(s)}{R(s)} = \frac{(abcde) + (abge)}{1 - (bj + gh + cdh + di + f) + bjdi + bjf}$$